0 Additional exercise

a. $\vec{A}\cdot(\vec{B}\wedge\vec{C}) = A_i(\vec{B}\wedge\vec{C})_i$

$= A_i\,\epsilon_{ijk}B_jC_k$ use permutation: $\epsilon_{ijk}=\epsilon_{kij}=\epsilon_{jki}$

$= A_i\cdot\epsilon_{kij}B_jC_k$ order of scalar product is arbitrary $= C_k\cdot\epsilon_{kij}A_i\cdot B_j$

$= C_k\cdot(\vec{A}\wedge\vec{B})_k = \vec{C}\cdot(\vec{A}\wedge\vec{B})$

$= A_i\cdot\epsilon_{jki}B_jC_k = B_j\,\epsilon_{jki}C_kA_i = B_j\cdot(C\wedge A)_j = \vec{B}\cdot(\vec{C}\wedge\vec{A})$

$= \vec{B}\cdot(\vec{C}\wedge\vec{A})$

b. $\vec{\nabla}\wedge\vec{\nabla}\psi = 0$: $\epsilon_{ijk}\,\partial/\partial x_j\cdot(\partial/\partial x_k\psi) = \partial/\partial x_j\cdot\partial/\partial x_k\psi - \partial/\partial x_k\cdot\partial/\partial x_j\psi$

$\nabla_i = \partial/\partial x_i$

$= 0.$ x_j and x_k are independent - so the order of the partial derivatives does not matter.

c. $\left[\vec{\nabla}\wedge(\vec{\nabla}\wedge\vec{A})\right]_i = \epsilon_{ijk}\cdot\partial/\partial x_j\cdot\left[\vec{\nabla}\wedge\vec{A}\right]_k$ $\qquad \epsilon_{klm}=\epsilon_{mkl}=\epsilon_{lmk}$

$= \epsilon_{klm}\cdot\partial/\partial_l\cdot A_m$

$= \epsilon_{ijk}\cdot\epsilon_{lmk}\cdot\partial/\partial x_j\cdot\partial/\partial_l\cdot A_m$

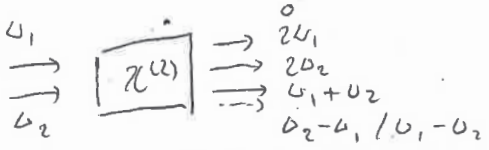
$= \left[\delta_{il}\cdot\delta_{jm} - \delta_{im}\cdot\delta_{jl}\right]\partial/\partial x_j\cdot\partial/\partial_l\cdot A_m$

$\delta_{il}\cdot\partial/\partial_l\,\partial/\partial x_j A_m\cdot\delta_{jm} - \delta_{jl}\cdot\partial/\partial x_j\cdot\partial/\partial_l\cdot A_m\cdot\delta_{im}$

$\left[\vec{\nabla}\cdot\vec{\nabla}\cdot\vec{A}\right]_i - \left[\nabla^2\vec{A}\right]_i$

Exercise 1:

Second-order:



possible frequencies.

→ see next page for work out.

All of these processes can be used in the generation of light but also to study materials.

SHG: Surface spectroscopy
Surface imaging or bulk imaging

SH scattering → particle interfaces
nanoscale structure in bulk liquids.

Mostly probe of electronic structure; resonant or non-resonant

SFG: Often used as surface vibrational spectroscopy
to measure a vibrational spectra of interfaces – planar or particle interfaces. Another SFG process is the linear electro optic effect.

SHG / SFG are useful because of their surface specificity. This property follows from the spatial symmetry properties of the material under study.

Third-order processes:

There are many third-order processes; some are used to generate new frequencies, others as switches or to manipulate light, to probe matter, or as a distortion to experiments.

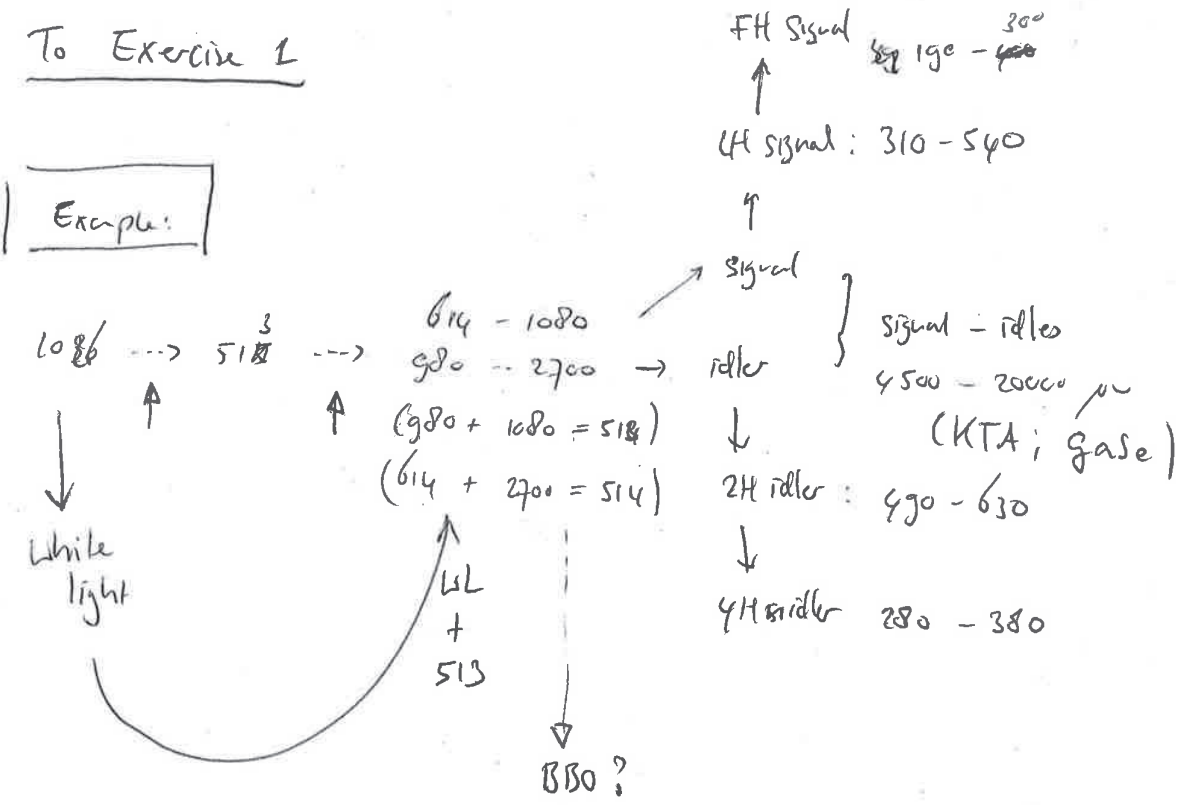
Exercise 1, continued:

Examples:

- Third harmonic generation
- Intensity dependent refractive index
- Kerr effect
- Electric field induced second harmonic generation.

To Exercise 1

Example:

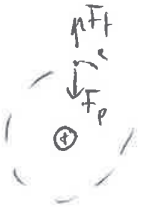


Exercise 2

2a.

Attraction of nucleus with electron (Coulomb): centripetal (F_p)

Reaction force due to rotating motion in fixed reference frame: centrifugal (F_f)



b. $m_e \cong 10^{-31}$ kg $\quad m_{H^+} \cong 10^{-27}$ kg - the electron moves, the nucleus remains fixed

c.

$\sin\theta = \frac{l}{R}$

$F_e = \frac{-e^2}{4\pi\varepsilon_0\varepsilon R^2}$

$F_{ind} = -e\cdot E = -\frac{e^2\cdot\sin\theta}{4\pi\varepsilon_0\varepsilon R^2} = \frac{-e^2\cdot l}{4\pi\varepsilon_0\varepsilon R^3} \Rightarrow \frac{e\cdot P_{ind}}{4\pi\varepsilon_0\varepsilon R^3} = eE$

$P_{ind} = -e\cdot l$

$= \bar{A}\,\kappa_0 E$

↑

e: -; E: -

$P_{ind} = \underbrace{4\pi\varepsilon_0\varepsilon R^3}_{\alpha_0}\cdot E$

d. We have $P^{(1)} = N\cdot\langle p^{(1)}\rangle$ for a gas this is simply $N\cdot p^{(1)}$

$P^{(1)} = \varepsilon_0\chi^{(1)}E = N\cdot p^{(1)} = N\cdot 4\pi\varepsilon_0\varepsilon R^3\cdot E$

$\Rightarrow \chi^{(1)} = 4\pi\varepsilon R^3\cdot N$ and $\varepsilon = 1+\chi^{(1)}$

N: molecules / m^3 $\quad R^3$: m^3

the gas

$d = 70.85\ \frac{kg}{m^3}$

$M = 2\ g/mol = 2\cdot10^{-3}\ kg/mol$

$N = 70.85\cdot 500\cdot 6.02\cdot10^{23} = 2.13\cdot10^{28}\ \frac{molec}{m^3}$

$\frac{kg}{m^3}\cdot\frac{mol}{kg}\cdot\frac{molec}{mol}$

$R^3 = (5.29\cdot10^{-11})^3 = 1.48\cdot10^{-31}\ m^3$

$\chi^{(1)} = 0.04$

$\varepsilon = 1.04.$

Exercise 3

3a. $E = \frac{e}{4\pi\varepsilon_0 \varepsilon R^2} \rightarrow E \simeq 5 \cdot 10^{11} \frac{V}{m}$

$a_0 = 5.29 \cdot 10^{-11}\ m$

b. $I = 2n\varepsilon_0 c |E|^2 = 1.3 \cdot 10^{17}\ W/cm^2$

c. $P_{peak} = \frac{P_{avg}}{f \cdot \tau}$ (rep rate ; puls dur) $\qquad \frac{P_{avg}}{f}$ = pulse energy

$I_{peak} = \frac{P_{peak}}{A} = \frac{P_{avg}}{f \cdot \tau \cdot A}$

↑ area

$0.013 \cdot f = 1$

$f = 77$ Hz ; so not with 1 kHz - 1 MHz.

(due to the small focus)

Note that with this type of ~~intensity~~ pulse energy (13 nJ) there is typically already white light generation with this narrow focus.